Time Constants
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Solutions to time dependent differential equations typically are comprised of two types: one that persists through time and one that dies off. The former is termed the steady state solution and may be time dependent. The latter is called the transient solution. It is typically of interest to determine how long it takes for the transient solution to die off. There is no particular time at which one can say the transient is zero, only that it is on its way to zero.

Most problems present a time that can be considered to be characteristic of the decay of the transient. This time is called a time constant. The EETs consider e^{-5} to be close enough to zero that the transient has died off and the steady state solution persists.

Examples:

Problem	Differential Equation	Formal Solution	Transient Solution	Steady State Solution	Time Constant
RC Circuit V =Constant	$RI+\frac{Q}{C}=V$	$Q=CV+(Q_0-CV)e^{-t/RC}$	$Q=(Q_0-CV)e^{-t/RC}$	$Q=CV$	$\tau=RC$
RL Circuit V =Constant	$L\frac{dI}{dt}+RI=V$	$I=\frac{V}{R}+\left(I_0-\frac{V}{R}\right)e^{-t/(L/R)}$	$I=\left(I_0-\frac{V}{R}\right)e^{-t/(L/R)}$	$I=\frac{V}{R}$	$\tau=\frac{L}{R}$
Terminal Velocity	$m\frac{dv}{dt}=mg-bv$	$v=\frac{mg}{b}+\left(v_0-\frac{mg}{b}\right)e^{-t/(m/b)}$	$v=\left(v_0-\frac{mg}{b}\right)e^{-t/(m/b)}$	$v=\frac{mg}{b}$	$\tau=\frac{m}{b}$