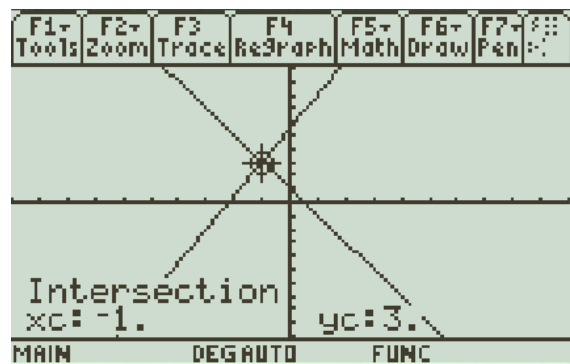


Solve:  $2x + y = 1$        $5x - 2y = -11$

Solve for y. We will need it.       $y = 1 - 2x$        $y = (5x + 11)/2$

**1) Graphing**

Graph the two equations. Make sure the intersection is visible. While looking at the graph, F5/5 (Intersection). First it asks for the two curves to intersect. The current curve number is in the upper right hand corner. Move from curve to curve using the up and down arrows. Press Enter when you are on a curve of interest. Lower bound is to the left of the intersection. Upper bound is to the right of the intersection.



**2) Substitution**

Solve one equation for a variable. Substitute it in the other equation. Solve the resulting equation. Solve for the other variable.

$$y = 1 - 2x$$

$$5x - 2(1 - 2x) = -11$$

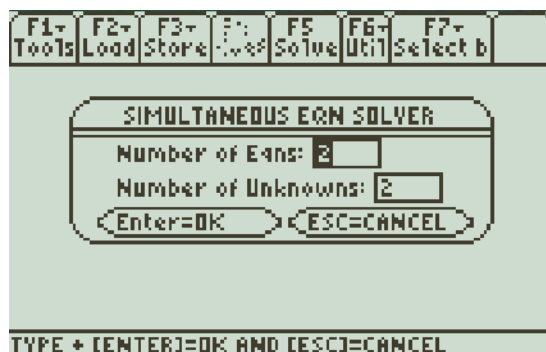
$$5x - 2 + 4x = -11$$

$$9x = -9$$

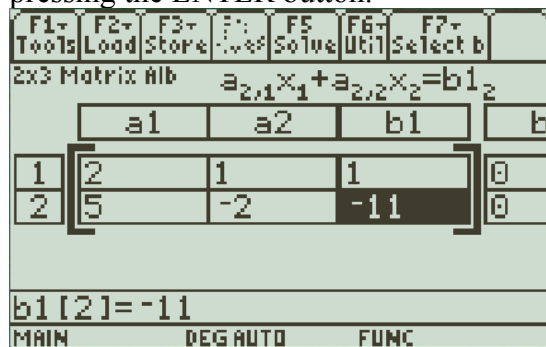
$$x = -1$$

$$y = 1 - 2(-1) = 3$$

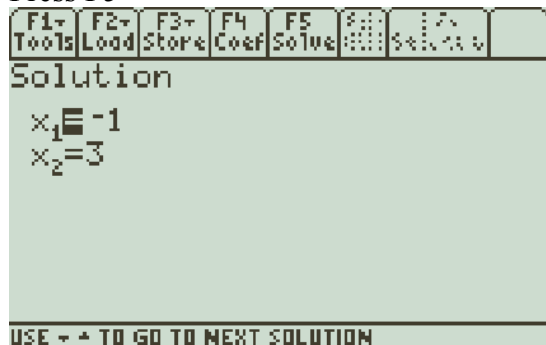
**5) Apps/FlashApps/Simultaneous Eqn Solver** (if no icons)  
**Apps/Simultaneous Eqn Solver** (if icons)  
Select *New*



Fill in the array by entering the number then pressing the ENTER button.



Press F5



Note that F4 allows you to reenter the coefficients. See what the other F keys let you do.

### 3) Addition/Subtraction

Line up the x's and y's. If necessary, multiply an equation first so that the coefficients agree. Add or subtract. Solve the resulting equation. Solve for the other variable.

$$2(2x + y = 1) \quad \text{so} \quad 4x + 2y = 2$$

$$5x - 2y = -11$$

$$4x + 2y = 2$$

Add the two equations.

$$5x - 2y + 4x + 2y = -11 + 2$$

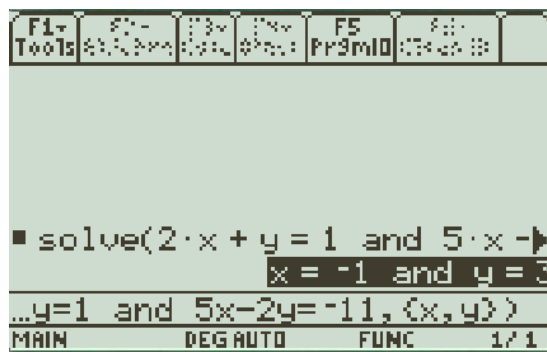
Simplify

$$9x = -9$$

$$x = -1$$

$$y = 1 - 2(-1) = 3$$

### 6) Solver



### 4) Cramer's Rule

**Determinant:**

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a)(d) - (b)(c)$$

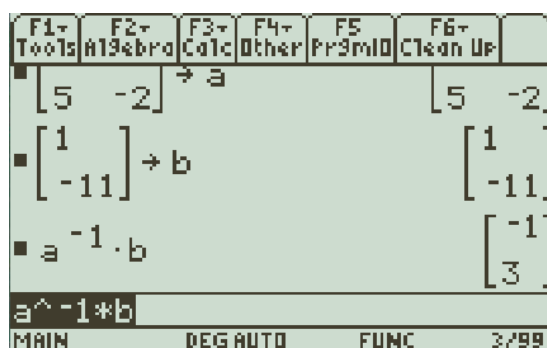
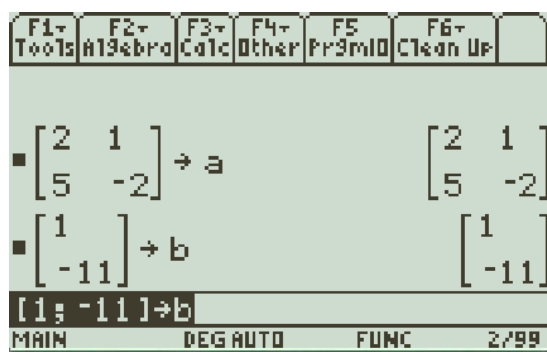
**Solution by Cramer's rule**

$$x = \frac{\begin{vmatrix} 1 & 1 \\ -11 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 5 & -2 \end{vmatrix}} = \frac{(1)(-2) - (1)(-11)}{(2)(-2) - (1)(5)} = \frac{9}{-9} = -1$$

$$y = \frac{\begin{vmatrix} 2 & 1 \\ 5 & -11 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 5 & -2 \end{vmatrix}} = \frac{(2)(-11) - (1)(5)}{(2)(-2) - (1)(5)} = \frac{-27}{-9} = 3$$

**Note:** same denominator, replace appropriate column by RHS of original equations. The pattern is easier to see in the 3x3 case below.

### 7) Matrix Inverse (Chapter 16)



Note that row elements are separated by commas and rows themselves, by semicolons. You must use square brackets for vectors. The matrix  $a$  is entered  $[2,1;5,-2]$ .

## NxN equations

All but graphing can be used to solve higher order equations. We look at a 3x3 case.

$$x + y + z = 2$$

**Solve:**  $x - z = 1$  for  $x, y,$  and  $z.$

$$x + y = 1$$

**4) Cramer's Rule** – we solve for  $x, y,$  and  $z.$  The determinants used in solving for  $z$  are shown. Note that they all have the same denominator so you only need to find it once.

$$x = \frac{\begin{vmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}} = \frac{2}{1} = 2 \quad y = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}} = \frac{-1}{1} = -1 \quad z = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}} = \frac{1}{1} = 1$$

Denominator:	Numerator:

## 6) Solver

`solve(x + y + z = 2 and x - z = 1 and x + y = 1, {x,y,z})`

	<p>Pretty print shows the beginning of the equation in the image to the left and the command line shows the rest of it.</p>
--	---

## 5) Simultaneous Eqn Solver

<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 12.5%;">F1- Tools</td> <td style="width: 12.5%;">F2- Load</td> <td style="width: 12.5%;">F3- Store</td> <td style="width: 12.5%;">F4- Vars</td> <td style="width: 12.5%;">F5- Solve</td> <td style="width: 12.5%;">F6- Util</td> <td style="width: 12.5%;">F7- Select b</td> </tr> <tr> <td colspan="7" style="text-align: center; padding: 10px;"> <p><b>SIMULTANEOUS EQN SOLVER</b></p> <p>Number of Eqs: <input type="text" value="3"/></p> <p>Number of Unknowns: <input type="text" value="3"/></p> <p>Enter=OK    &lt;ESC=CANCEL&gt;</p> </td> </tr> <tr> <td colspan="2">MAIN</td> <td colspan="2">DEG AUTO</td> <td colspan="3">FUNC</td> </tr> </table>	F1- Tools	F2- Load	F3- Store	F4- Vars	F5- Solve	F6- Util	F7- Select b	<p><b>SIMULTANEOUS EQN SOLVER</b></p> <p>Number of Eqs: <input type="text" value="3"/></p> <p>Number of Unknowns: <input type="text" value="3"/></p> <p>Enter=OK    &lt;ESC=CANCEL&gt;</p>							MAIN		DEG AUTO		FUNC			<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 12.5%;">F1- Tools</td> <td style="width: 12.5%;">F2- Load</td> <td style="width: 12.5%;">F3- Store</td> <td style="width: 12.5%;">F4- Vars</td> <td style="width: 12.5%;">F5- Solve</td> <td style="width: 12.5%;">F6- Util</td> <td style="width: 12.5%;">F7- Select b</td> </tr> <tr> <td colspan="7" style="padding: 5px;"> <p>3x4 Matrix A b    <math>a_{1,1}x_1 + \dots + a_{1,3}x_3 = b_1</math></p> <table border="1" style="width: 100%; border-collapse: collapse; margin: 5px;"> <tr> <td style="width: 10%;"></td> <td style="width: 20%;">a1</td> <td style="width: 20%;">a2</td> <td style="width: 20%;">a3</td> <td style="width: 30%;">b1</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>1</td> <td>0</td> <td>-1</td> <td>1</td> </tr> <tr> <td>3</td> <td>1</td> <td>1</td> <td>0</td> <td>1</td> </tr> </table> <p>a(1,1)=1</p> </td> </tr> <tr> <td colspan="2">MAIN</td> <td colspan="2">DEG AUTO</td> <td colspan="3">FUNC</td> </tr> </table>	F1- Tools	F2- Load	F3- Store	F4- Vars	F5- Solve	F6- Util	F7- Select b	<p>3x4 Matrix A b    <math>a_{1,1}x_1 + \dots + a_{1,3}x_3 = b_1</math></p> <table border="1" style="width: 100%; border-collapse: collapse; margin: 5px;"> <tr> <td style="width: 10%;"></td> <td style="width: 20%;">a1</td> <td style="width: 20%;">a2</td> <td style="width: 20%;">a3</td> <td style="width: 30%;">b1</td> </tr> <tr> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>2</td> </tr> <tr> <td>2</td> <td>1</td> <td>0</td> <td>-1</td> <td>1</td> </tr> <tr> <td>3</td> <td>1</td> <td>1</td> <td>0</td> <td>1</td> </tr> </table> <p>a(1,1)=1</p>								a1	a2	a3	b1	1	1	1	1	2	2	1	0	-1	1	3	1	1	0	1	MAIN		DEG AUTO		FUNC		
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USE ← → TO GO TO NEXT SOLUTION					

## 7) Matrix Inverse (Chapter 16)

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