MTH 352 **Exam 3** **December 2009**

Prof. Townsend

Name <u>Solutions</u>

This exam covers the material in exams 1 and 2. Do not use Laplace transforms on this exam. You may use deSolve to check your work but not for the problem solution itself. You may use your calculator for all algebra, integrals, and derivatives.

1) (25 points) Solve one the following differential equations with $y(0) = -4$ using linear techniques.	$\frac{dy}{dx} - xy = -3x$ $p = -x$ $I = e^{-x^2/2}$ $q = -3x$ $y = e^{x^2/2}\left\{C - 3\int e^{-x^2/2} x\,dx\right\}$ $y = e^{x^2/2}\left\{C + 3e^{-x^2/2}\right\}$ $y = Ce^{x^2/2} + 3$ $-4 = C + 3 \qquad C = -7$ $y = 3 - 7e^{x^2/2}$	$\frac{dy}{dx} - 3y = 2$ $p = -3$ $I = e^{-3x}$ $q = 2$ $y = e^{3x}\left\{C + 2\int e^{-3x}\,dx\right\}$ $y = e^{3x}\left\{C - \frac{2}{3}e^{-3x}\right\}$ $y = Ce^{3x} - \frac{2}{3}$ $-4 = C - \frac{2}{3} \qquad C = -\frac{10}{3}$ $y = \frac{2}{3} - \frac{10}{3}e^{3x}$

2) (25 points) Solve one the following differential equations with $y(0) = -3$ using separation of variables.	$\frac{dy}{dx} = \frac{x^2+4}{y}$ $ydy = \left(x^2+4\right)dx$ $\frac{y^2}{2} = \frac{x^3}{3} + 4x + C$ $y = \pm\sqrt{2\frac{x^3}{3} + 8x + C}$ $-3 = -\sqrt{C}$ $9 = C$ $y = -\sqrt{\frac{2}{3}x^3 + 8x + 9}$	$\frac{dy}{dx} = y^2x^4$ $\frac{dy}{y^2} = x^4dx$ $y^{-2}dy = x^4dx$ $-y^{-1} = \frac{1}{5}x^5 + C$ $y = \frac{-5}{x^5+C}$ $-3 = \frac{-5}{C}$ $\quad C = \frac{5}{3}$ $y = \frac{-15}{3x^5+5}$
3) (25 points) Solve one the following differential equations with $y(0) = 1$ and $y'(0) = 0$.	$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 6y = 0$ $\lambda^2 + 4\lambda + 6 = 0$ $\lambda = -2 \pm \sqrt{2}i$ $y = e^{-2x}\left\{A\cos\left(\sqrt{2}x\right) + B\sin\left(\sqrt{2}x\right)\right\}$ $y' = -e^{-2x}\sqrt{2}\left\{\left(2A - \sqrt{2}B\right)\cos\left(\sqrt{2}x\right) + \right.$ $\left. + \left(2B + \sqrt{2}A\right)\sin\left(\sqrt{2}x\right)\right\}$ $1 = A$ $0 = -\sqrt{2}\left(2A - \sqrt{2}B\right)$ $A = 1$ $\quad B = \sqrt{2}$ $y = e^{-2x}\left\{\cos\left(\sqrt{2}x\right) + \sqrt{2}\sin\left(\sqrt{2}x\right)\right\}$	$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$ $\lambda^2 - 6\lambda + 9 = 0$ $(\lambda - 3)^2 = 0$ $y = e^{3x}\{A + Bx\}$ $y' = e^{3x}\{3A + B + 3Bx\}$ $1 = A$ $0 = 3A + B$ $A = 1$ $\quad B = -3$ $y = e^{3x}\{1 - 3x\}$

4) (25 points) Solve one the following differential equations with $y(0)=1$ and $y'(0)=0$.

$$W(x)=\begin{vmatrix} y_1(x) & y_2(x) \\ \dfrac{dy_1(x)}{dx} & \dfrac{dy_2(x)}{dx} \end{vmatrix}$$

$$W(x)=y_1(x)\frac{dy_2(x)}{dx}-y_2(x)\frac{dy_1(x)}{dx}$$

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=e^{-4x}$$

$$y_1(x)=e^{-x}$$

$$y_2(x)=e^{2x}.$$

$$W(x)=e^{-x}\left(2e^{2x}\right)-e^{2x}\left(-e^{-x}\right)=3e^{x}$$

$$u_1(x)=-\int\frac{e^{2x}e^{-4x}}{3e^{x}}dx=\frac{1}{9}e^{-3x}$$

$$u_2(x)=\int\frac{e^{-x}e^{-4x}}{3e^{x}}dx=-\frac{1}{18}e^{-6x}$$

$$y_p(x)=\frac{1}{9}e^{-3x}e^{-x}-\frac{1}{18}e^{-6x}e^{2x}=\frac{1}{18}e^{-4x}$$

$$y(x)=C_1e^{-x}+C_2e^{2x}+\frac{1}{18}e^{-4x}$$

$$y'(x)=-C_1e^{-x}+2C_2e^{2x}-\frac{4}{18}e^{-4x}$$

$$1=C_1+C_2+\frac{1}{18}$$

$$0=-C_1+2C_2-\frac{4}{18}$$

Solver: $C_1=\frac{5}{9}$ $\qquad C_2=\frac{7}{18}$

$$y(x)=\frac{5}{9}e^{-x}+\frac{7}{18}e^{2x}+\frac{1}{18}e^{-4x}$$

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+8y=7$$

$$y_1(x)=e^{-2x}\cos(2x)$$

$$y_2(x)=e^{-2x}\sin(2x).$$

Define y1 and y2 in the graph menu then enter

$$W(x)=y_1(x)\frac{dy_2(x)}{dx}-y_2(x)\frac{dy_1(x)}{dx}$$

you get

$$W(x)=2e^{-4x}$$

$$u_1(x)=-7\int\frac{e^{-2x}\sin(2x)}{2e^{-4x}}dx$$

$$u_2(x)=7\int\frac{e^{-2x}\cos(2x)}{2e^{-4x}}dx$$

$$u_1(x)=7\frac{e^{2x}\{\cos(2x)-\sin(2x)\}}{8}$$

$$u_2(x)=7\frac{e^{2x}\{\cos(2x)+\sin(2x)\}}{8}$$

$$y_p(x)=y_1(x)u_1(x)+y_2(x)u_2(x)=\frac{7}{8}$$

$$y(x)=C_1e^{-2x}\cos(2x)+C_2e^{-2x}\sin(2x)+\frac{7}{8}$$

$$y'(x)=-2e^{-2x}\left\{(C_1-C_2)\cos(2x)+(C_1+C_2)\sin(2x)\right\}$$

$$1=C_1+\frac{7}{8}$$

$$0=-2(C_1-C_2)$$

		$C_1 = C_2 = \frac{1}{8}$ $y(x) = \frac{1}{8}\left\{e^{-2x}\cos\left(2x\right) + e^{-2x}\sin\left(2x\right) + 7\right\}$