

Name _____

You may use your calculator, your class notes, your book, and your brain as sources of information. You may not share information or materials with others. All telecommunications devices must be turned off.

- 1) Show **ALL** work. Give me something to grade. You must solve the problems using the rules given in class, not just using the TI-89.
- 2) You may use your TI-89 for algebra and for checking your work. Please write down any calculator steps you use so that I could repeat your steps on my own calculator.
- 3) Provide at least three (3) significant digits for all non-integers.
- 4) You will receive extra credit for checking your work properly.
- 5) All problems bear the same weight.

1) Use the table method *or* the delta method to find the slope of the line tangent to the curve $y = 3x(1 - 5x)$ when $x=3$. If using the table method, choose $x=2.9, 2.99, 2.999, 2.9999$.

$$m = \frac{f(x+h) - f(x)}{h} = \frac{3(x+h)(1-5(x+h)) - 3x(1-5x)}{h}$$

$$= \frac{-30h^2x - 3h(5h-1)}{h} = \frac{-30h^2x - 15h^2 + 3h}{h} = -30x - 15h + 3$$

Let $h \rightarrow 0$, then $x=3$: $m \rightarrow -30x + 3 \rightarrow -87$

$x = 3, y = -126$			$m \rightarrow -3(10x - 1)$	$m \rightarrow -87$
x	2.9	2.99	2.999	2.9999
y	-117.45	-125.1315	-125.913	-125.9913
Δx	0.1	0.01	0.001	0.0001
Δy	-8.55	-0.8685	-0.086985	-0.00869985
$m = \frac{\Delta y}{\Delta x}$	-85.5	-86.85	-86.99	-87

Note: for the rest of the problems, use derivative rules, *not* the delta method.

Once you have applied all rules of derivatives to determine $\frac{dy}{dx}$, it is not necessary to manipulate the result further.

2) Find the slope of the line tangent to the curve defined by $y = 5x^2 - x - 7$ when $x=3$.

$$y' = 10x - 1 \rightarrow 29$$

To find equation of the tangent line:

$$x=3 \rightarrow y=35$$

$$y = mx + b$$

$$35 = 29 * 3 + b$$

$$b = -52$$

$$y = 29x - 52$$

3) Find $\frac{dy}{dx}$ for $y = 7x^5 - \sqrt{x} - 7$. Identify the $\frac{dy}{dx}$ rule used for each step

Rules used: $\sqrt{x} = x^{\frac{1}{2}}$

Eq. #	Rule
23.8	$\frac{dc}{dx} = 0$
23.10	$\frac{d(cu)}{dx} = c \frac{du}{dx}$
23.9	$\frac{dx^n}{dx} = nx^{n-1}$
23.11	$\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

$$y = 7x^5 - \sqrt{x} - 7 \quad y = 7x^5 - x^{\frac{1}{2}} - 7$$

$$y' = 35x^4 - \frac{1}{2}x^{-\frac{1}{2}} = 35x^4 - \frac{1}{2\sqrt{x}}$$

4) Find $\frac{dy}{dx}$ for $y = (x-1)(x^3+1)^9$

$$y' = (x^3+1)^9 + 9(x-1)(x^3+1)^8(3x^2)$$

$$\text{Factor: } y' = (x^3+1)^8 \{ (x^3+1) + 9(x-1)(3x^2) \} = (x^3+1)^8 \{ 28x^3 - 27x^2 + 1 \}$$

5) Find $\frac{dy}{dx}$ for $y = \frac{x-1}{x^3+1}$

$$u = x-1 \quad u' = 1 \quad v = x^3+1 \quad v' = 3x^2$$

$$y' = \frac{(x^3+1) \cdot 1 - (x-1)(3x^2)}{(x^3+1)^2}$$

Expand the numerator

$$y' = -\frac{(2x^3 - 3x^2 - 1)}{(x^3+1)^2}$$

6) Find $\frac{d^2y}{dx^2}$ for $y = 3x^7 - 2x^3 + 8$

$$y' = 21x^6 - 6x^2$$

$$y'' = 126x^5 - 12x$$

TI: $d(3x^7-2x^3+8, x, 2)$

Extra Credit) Find $\frac{dy}{dx}$ for in terms of x and y using the method of implicit derivatives.

Analytical – 10 points , TI-89 only – 3 points.

$$x^2y - 3xy^5 = 7 \quad y' = \frac{-(2x - 3y^4)y}{x(x - 15y^4)}$$

$$2xy + x^2 \frac{dy}{dx} - 3y^5 - 15xy^4 \frac{dy}{dx} = 0$$

$$(2xy - 3y^5) + (x^2 - 15xy^4) \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{(2xy - 3y^5)}{(x^2 - 15xy^4)}$$

TI: $\text{ImpDif}(x^2*y-3x*y^5=7, x, y)$