Fall 2006 MU solutions

1) $y = 2\sqrt{x} - x + 3$

Tangent slope is first derivative of y

$\sqrt{x} = x^{\frac{1}{2}}$ $\frac{d}{dx}\sqrt{x} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$y' = \frac{1}{\sqrt{x}} - 1$

$m = \frac{1}{\sqrt{4}} - 1 = -\frac{1}{2}$ first derivative of y evaluated at x=4.

There are two ways to find the equation of the tangent line

i) $3 = -\frac{1}{2}(4) + b$ $b = 5$

ii) $(y-3) = -\frac{1}{2}(x-4)$ from $(y - y_1) = m(x - x_1)$

both give $y = -\frac{1}{2}x + 5$ Answer is A

2) $y = (4x-7)^3$

For either the tangent or normal line you need to first find the first derivative of y.

$y' = 3(4x-7)^2 4 = 12(4x-7)^2$ by equation (23.15)

The slope of the normal is given as –1/12. Hence the slope of the tangent line is 12.

$12 = 12(4x-7)^2$ → $x = 2$ → $y = (4*2-7)^3 = 1$

The equation of the normal line is found from $1 = -\frac{1}{12}(2) + b$ → $b = \frac{7}{6}$

$y = -\frac{1}{12}x + \frac{7}{6}$ multiply by 12: $12y = -x + 14$

Rewrite $12y + x - 14 = 0$ Answer is B

3) $V = I * R$ V is constant.

Take the derivative: $0 = I\frac{dR}{dt} + \frac{dI}{dt}R$

Solve for $\frac{dR}{dt}$ $\frac{dR}{dt} = -\frac{dI}{dt}\frac{R}{I}$ (note – Units say this is OK)

Plug in numbers $\frac{dR}{dt} = -(-4)\frac{R}{32}$

Find R from V=12 $R = \frac{V}{I} = \frac{12}{32} = \frac{3}{8}$

Plug in $\frac{dR}{dt} = -(-4)\frac{1}{32}\frac{3}{8} = \frac{3}{64}$ Answer is C

4) $h = 12t^3 - 198t^2 + 9500$

Turn up in dive means find the minimum.

$h' = 36t^2 - 2*198t = 36t^2 - 396t = 36t(t-11)$

$h'' = 72t - 396 = 36(2t-11)$

$h' = 0$ for $t = 0,11$

$h'' = 36(0-11), 36(22-11)$

$h'' > 0$ for $t = 11$

Reread the problem. We want the altitude.

$h = 12(11)^3 - 198(11)^2 + 9500 = 1514$ Answer is D

Note – define y1=12x^3-198x^2+9500. Then you can

i) graph it – since y=9500 at t=0 start with ymin=0, ymax=10000 xmin=0, xmax=10 (zoom standard value). You don't see the maximum so try xmax=20. That works. Then you can find the minimum using F5 to check your answer for both where the minimum is and that it the curve is concave up there.

ii) find d(y1(x),x) the first derivative

iii) solve(d(y1(x),x)=0,x) find the minimum

iv) find d(y1(x),x,2) the second derivative

5) Girth condition: $L + 4W \le 96.$ $V = L*W^2$

Assume $L + 4W = 96.$

Then $V = (96-4W)W^2 = 96W^2 - 4W^3$

Find the first derivative of the volume

$V' = 192W - 12W^2 = 12W(16-W)$

$V'' = 192 - 24W = 24(8-W)$

Find the minimum. W=0 gives a minimum. W=16 gives a maximum as $V'' = 24(8-16) < 0$

We have W=16. We need L. $L = 96 - 4W = 96 - 4*16 = 32$

The box is 16x16x32. Answer is C

Fall 2010 MU solutions

1) $x = t + \frac{-2}{t}$ $\quad y = t - \frac{-2}{t}$

To find the acceleration you first need velocity so find both second derivatives.

$$x = t + \frac{-2}{t} = t - 2t^{-1} \qquad y = t - \frac{-2}{t} = t + 2t^{-1}$$

$$v_x = \frac{dx}{dt} = 1 + 2t^{-2} \qquad v_y = \frac{dy}{dt} = 1 - 2t^{-2}$$

$$a_x = \frac{dv_x}{dt} = -4t^{-3} \qquad a_y = \frac{dv_y}{dt} = 4t^{-3}$$

$$\vec{a} = \frac{4}{7^3}[-1,1]$$

At t=7 $\quad |\vec{a}| = \frac{4}{7^3}\sqrt{2} = 0.0165$

$$\theta_{calculator} = \tan^{-1}\left(\frac{1}{-1}\right) = -45^\circ$$

Since the x component is negative and y component is positive, the acceleration is in quadrant II. $\quad \theta_{acc} = 180^\circ - 45^\circ = 135^\circ$

Answer is B. I disagree with the test bank whose angle is $\theta_{acc} = 360^\circ - 45^\circ = 315^\circ$, which is in quadrant IV. Who's right? I checked by using

$d\left(\left[t + \frac{-2}{t}, t + \frac{-2}{t}\right], t, 2\right) | t = 7$ $\quad$ to get $\quad \left[-\frac{4}{343}, \frac{4}{343}\right]$

then $\left[-\frac{4}{343}, \frac{4}{343}\right] \triangleright polar = [0.0165\angle 135]$ $\quad$ so I think I'm right.

Note – there will be times when the answers to the Makeup Exams are incorrect. It is rare but it happens. Please let me know if you think this case has occurred.

2) $x = 8t^2 + 6t + 2$ $\quad y = -6t^2 - 5t - 6$

Velocity is first derivative.

$$x = 8t^2 + 6t + 2 \qquad y = -6t^2 - 5t - 6$$

$$v_x = \frac{dx}{dt} = 16t + 6 \qquad v_y = \frac{dy}{dt} = -12t - 5$$

$$v_x |_{t=8} = 134 \qquad v_y |_{t=8} = -101$$

Quadrant IV so calculator angle is fine.

$$\vec{v} = [134, -101] \triangleright polar = [167.8\angle -37]$$

or $|\vec{v}| = \sqrt{134^2 + (-101)^2} = \sqrt{28157} = 167.8$,

$$\theta_{calculator} = \tan^{-1}\left(\frac{-101}{134}\right) = -37^\circ = 360^\circ - 37^\circ = 323^\circ$$ Answer is A.

Note: there could also be a problem like example 4 where I give y(x) along with v_x.

3) $y = 120$ $v_x = 7$ $v_y = 0$ (it's moving horizontally, not vertically)

z is distance to kite. Find $\frac{dz}{dt}$ when $z = 130$.

Use Pythagorean Theorem.

$$z^2 = x^2 + y^2$$

$$\frac{dz}{dt} = \frac{1}{z}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

Solve for x. $x = \sqrt{z^2 - y^2} = \sqrt{130^2 - 120^2} = 50$

Now find $\frac{dz}{dt}$.

$$\frac{dz}{dt} = \frac{1}{130}(50 * 7 + 0) = 2.7 \text{ m/s}$$ Answer is B

4) Find $\frac{dr}{dt}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$ Answer is C

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt} = \frac{1}{4\pi 4^2} 80\pi = 1.25$$

5) $r = \frac{x}{2\pi}$ $V = \pi r^2 y = \pi\left(\frac{x}{2\pi}\right)^2 y = \frac{1}{4\pi}x^2 y$

$$P = 2x + 2y$$

Solve for y (so you don't have to square x)

$$V=\frac{1}{4\pi}x^2\left(\frac{P}{2}-x\right)=\frac{1}{8\pi}\left(Px^2-2x^3\right)$$
$$\frac{dV}{dx}=\frac{1}{8\pi}\left(2Px-6x^2\right)=\frac{1}{4\pi}x\left(P-3x\right)=0$$
$$x=\frac{P}{3}=\frac{24}{3}=8$$
$$y=\frac{24}{2}-8=4$$

Answer is A

6) $R=4x$ $\qquad C=0.01x^2+0.8x+90$

Maximize profit means maximize Revenue-Cost

$$P=R-C=4x-\left(0.01x^2+0.8x+90\right)=-0.01x^2+3.2x-90$$

$$\frac{dP}{dx}=-0.02x+3.2=0$$
$$x=\frac{3.2}{0.02}=160$$
$$\frac{d^2P}{dx^2}=-0.02<0,\Rightarrow \max$$

Answer is A

7) $x^2-y^2=17$

Need to first find $\frac{dy}{dx}$, then find negative reciprocal to find the slope of the normal.

$$2x-2y\frac{dy}{dx}=0$$
$$\frac{dy}{dx}=\frac{x}{y}=\frac{9}{8}$$
$$m_\perp=-\frac{8}{9}$$

Find b:

$$8=-\frac{8}{9}(9)+b$$
$$b=16$$
$$y=-\frac{8}{9}x+16$$
$$9y+8x-9*16=0$$
$$9y+8x-144=0$$

Answer is B