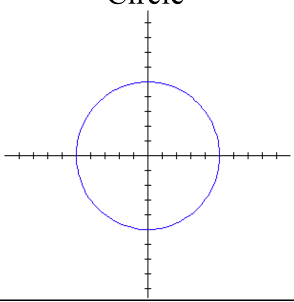
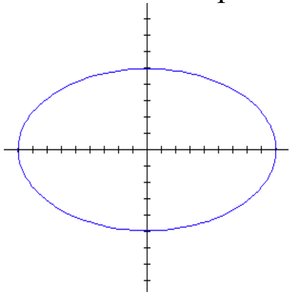
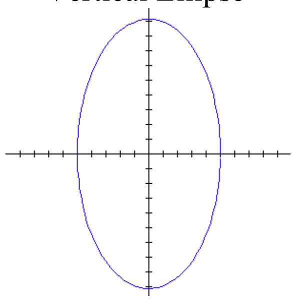
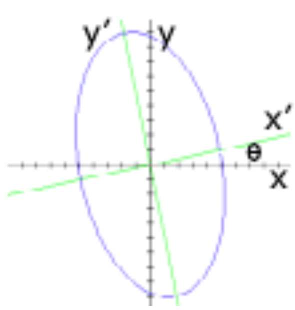
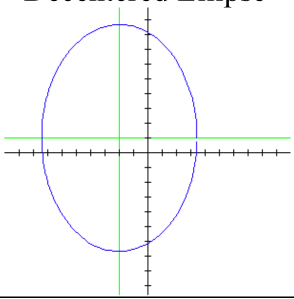
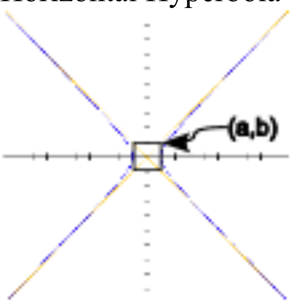
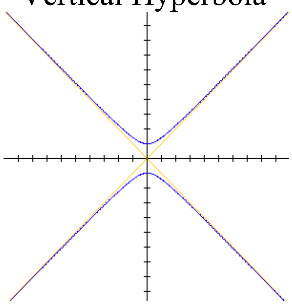
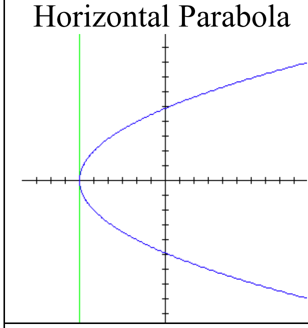
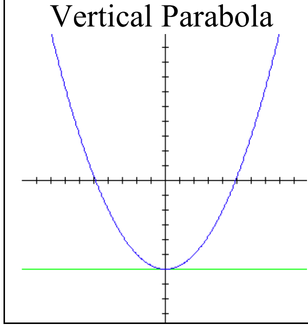
Prof. Townsend

$$Ax^2 + Bxy + Cy^2 + Dx + Cy + F = 0$$

Graphics generated from the following web site:
http://cs.jsu.edu/mcis/faculty/leathrum/Mathlets/awl/conics-main.html

Graph	A	B	C	D	E	F	h	k	Equation
Circle 	1.0	0	1.0	0	0	-25.0	0	0	$x^2 + y^2 = r^2$
Horizontal Ellipse 	0.3	0	1.0	0	0	-25.0	0	0	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a > b$ $a^2 = b^2 + c^2$ Vertices: $(\pm a, 0)$ Foci: $(\pm c, 0)$ $B^2 - 4AC < 0$
Vertical Ellipse 	1.0	0	0.3	0	0	-25.0	0	0	$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ $a > b$ $a^2 = b^2 + c^2$ Vertices: $(0, \pm a)$ Foci: $(0, \pm c)$ $B^2 - 4AC < 0$
Rotated Ellipse 	1.0	0.3	0.3	0	0	-25.0	0	0	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $x = x'\cos\theta - y'\sin\theta$ $y = x'\sin\theta + y'\cos\theta$ $\tan 2\theta = \frac{B}{A - C}$ if $A = C$ then $\theta = 45^\circ$

Decentered Ellipse 	1.0	0	0.5	4.0	-1.0	-25.0	-2.0	1.0	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
Horizontal Hyperbola 	1.0	0	-1.0	0	0	-1.0	0	0	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $c^2=a^2+b^2$ Vertices: $(\pm a,0)$ Foci: $(\pm c,0)$ Asymptotes: $y=\pm\frac{b}{a}x$ $B^2-4AC>0$
Vertical Hyperbola 	-1.0	0	1.0	0	0	-1.0	0	0	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ Vertices: $(0,\pm a)$ Foci: $(0,\pm c)$ Asymptotes: $y=\pm\frac{a}{b}x$ $B^2-4AC>0$
Horizontal Parabola 	0	0	1.0	-4.0	0	-24.0	0	-6.0	$y^2=4p(x-h)$ Standard Form: $y^2=4px$ Focus: $(p,0)$ Directrix: $x=-p$ $B^2-4AC=0$
Vertical Parabola 	1.0	0	0	0	-4.0	-24.0	0	-6.0	$x^2=4p(y-k)$ Standard Form: $x^2=4py$ Focus: $(0,p)$ Directrix: $y=-p$ $B^2-4AC=0$

Some notes on conics for class **MTH 232**

1) **Rotation** of the conic has occurred when the x's and y's are mixed up i.e. $B \neq 0$

2) **Translation** of the conic has occurred when $D \neq 0$ and/or $E \neq 0$ but $A \neq 0$ and/or $C \neq 0$

3) **Hyperbolas**. P 584 example 1, 2, 3

Notes: Assume the $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ form

a) b came out of the derivation. It is defined by $(0, \pm c)$

b) $y = 0$ gives $x = \pm a$ (the vertices)

c) $|x| \geq a$ or the equation does not work

d) for large y, $\frac{x^2}{a^2} = 1 + \frac{y^2}{b^2} \approx \frac{y^2}{b^2}$ hence the asymptotes are defined by $\frac{x^2}{a^2} = \frac{y^2}{b^2}$

e) The decentered form is $\frac{(x-h)^2}{a^2} + \sigma \frac{(y-k)^2}{b^2} = 1$

$$\text{where} \qquad \sigma = \begin{cases} 1 & \textit{ellipse} \\ -1 & \textit{hyperbola} \end{cases}$$

Decentration is just a shift of the origin of coordinates.

f) Expanded and written in standard form (e) gives

$$b^2x^2 + a^2y^2 - 2b^2hx - 2\sigma a^2ky + b^2h^2 + \sigma a^2k^2 - a^2b^2 = 0$$

$A = b^2$ $\quad B = 0$ $\quad C = a^2$ $\quad D = -2b^2h$

$E = -2\sigma a^2 k$ $\quad F = b^2h^2 + \sigma a^2k^2 - a^2b^2$

Similar notes apply for $\frac{y^2}{a^2} + \sigma \frac{x^2}{b^2} = 1$

Problems: p 587, #3, 11, 17, 21

4) **Parabolas**. P 574 example 2, 3

Notes: Assume the $y^2 = 4px$ form

a) The graph is horizontal since there are two y values for every x value.

b) The focus is on the x axis at (p,0)

c) The directrix is a line with constant x, $x = -p$.

d) The decentered form is $(y-k)^2 = 4p(x-h)$

Similar notes apply for $(x-h)^2 = 4p(y-k)$

Problems: p 576, #5, 13, 17, 19