Laplace and Z Transforms for Causal Functions

f(t) $t \geq 0$ *(causal)*	***F(s)***	***F(z)*** *(t=kT, T = Sample Time, k = Index)*
$d\ (t)$	1	$1 = z^{-0}$
$d\ (t - kT)$	e^{-kTs}	z^{-k}
$u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
t	$\frac{1}{s^2}$	$\frac{T \cdot z}{(z-1)^2}$
$\frac{t^k}{k!}$	$\left(\frac{1}{s}\right)^{k+1}$	$\underset{a \to 0}{Limit} \frac{(-1)^k}{k!} \cdot \frac{¶^k}{¶ a^k} \left(\frac{z}{z - e^{-aT}}\right)$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z - e^{-aT}}$
$t \cdot e^{-at}$	$\frac{1}{(s+a)^2}$	$\frac{T \cdot z \cdot e^{-aT}}{(z - e^{-aT})^2}$
$e^{-at} \cdot \cos(bt)$	$\frac{(s+a)}{(s+a)^2 + b^2}$	$\frac{z \cdot (z - e^{-aT} \cdot \cos(bt))}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$
$e^{-at} \cdot \sin(bt)$	$\frac{b}{(s+a)^2 + b^2}$	$\frac{z \cdot e^{-aT} \cdot \sin(bt)}{z^2 - z \cdot (2 \cdot e^{-aT} \cdot \cos(bt)) + e^{-2aT}}$

Initial Conditions

Laplace

$L\{f(t)\} = F(s)$

$L\{\dot{f}(t)\} = s \cdot F(s) - f(0)$

$L\{\ddot{f}(t)\} = s^2 \cdot F(s) - s \cdot f(0) - \dot{f}(0)$

$L\{\dddot{f}(t)\} = s^3 \cdot F(s) - s^2 \cdot f(0) - s \cdot \dot{f}(0) - \ddot{f}(0)$

Z

$Z\{f(k)\} = F(z)$

$Z\{f(k+1)\} = z \cdot F(z) - z \cdot f(0)$

$Z\{f(k+2)\} = z^2 \cdot F(z) - z^2 \cdot f(0) - z \cdot f(1)$

$Z\{f(k+3)\} = z^3 \cdot F(z) - z^3 \cdot f(0) - z^2 \cdot f(1) - z \cdot f(2)$

System Response using Laplace or Z transform

Given: Transfer function, *T(E)* or *T(P),* input, *x(k)* or *x(t),* output, *y(k)* or *y(t),* and initial conditions

1. Form the differential or difference equation relating *y* to *x*.
2. Take Z or Laplace transform of each term, substitute initial conditions and input.
3. If discrete, solve for $\frac{Y(z)}{z}$, if continuous solve for *Y(s)*. Partial fraction the right hand side.
4. If discrete, first solve for *Y(z)* by multiplying each partial fraction term by *z*. Take the and take the inverse Z or Laplace transform of each term using the table to solve for *y(k)* or *y(t).*